Dynamic Latent Space Model on Directed Networks

Author: Yunran Chen and Alexander Volfovsky

MOTIVATION

To better understand the underlying mechanism which drives the dynamic evolution and formation of the dynamic directed network.

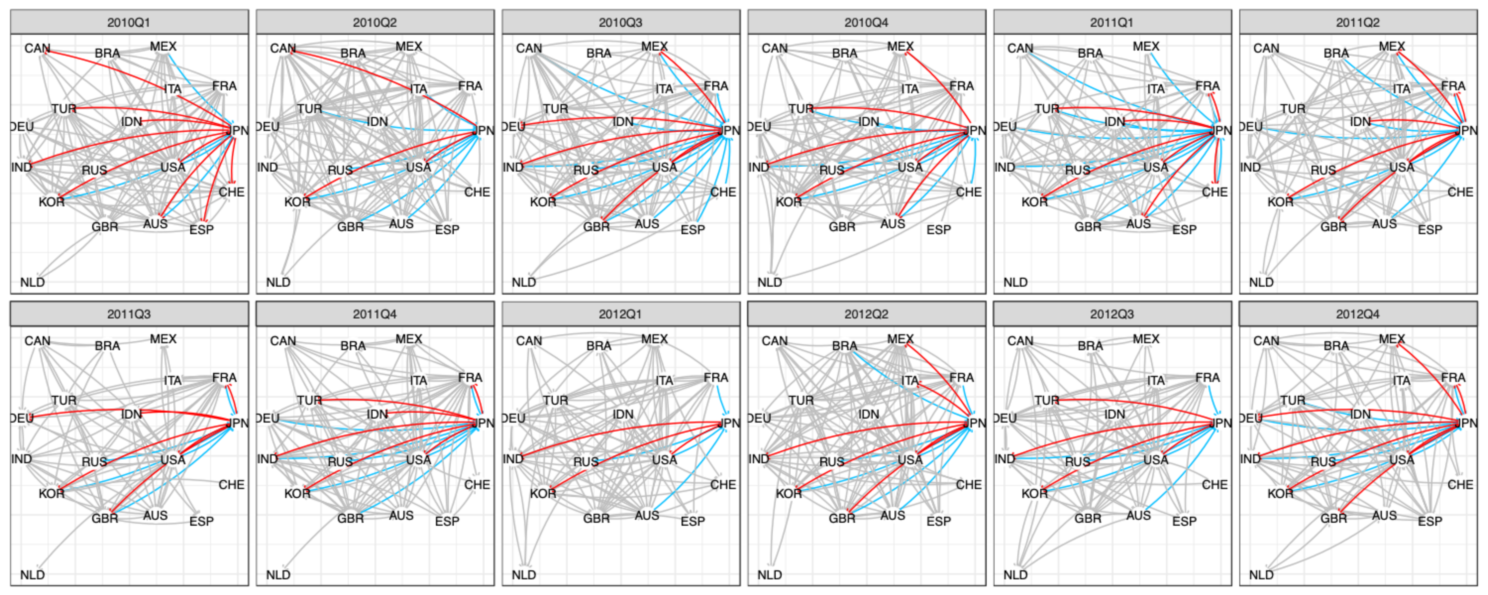

Fig 1 Dynamic visiting activities across the world's top 18 largest economies. Each edge represents at least one visit within the specified quarter.

- A trend of visiting activities among countries.
- Different countries clearly played distinct roles when interacting with each other.
- Several countries may show similar visiting activities, indicating the existence of stochastic equivalence and homophily.
- Bilateral relationships from an individual's aspect: solid allies, reciprocation across time, etc.

MODEL

Model Formulation

Inspired by Nonparametric Bayes model (Durante & Dunson, 2014) and Addictive and Multiplicative Model (Hoff , 2015) , we consider:

$$y_{ij}(t)|\pi_{ij}(t) \sim Bernoulli\{\pi_{ij}(t)\}$$

$$\pi_{ij}(t) = \{1 + e^{-S_{ij}(t)}\}^{-1}$$

$$S_{ij}(t) = \mu(t) + Z_{ij,t}^T \beta(t) + x_i^{sT}(t) x_j^r(t) + a_i(t) + b_j(t).$$

- Interpretable: 'sociability', 'popularity', latent space
- Well-performed: capture both homophily and stochastic equivalence, attributed to a third order dependence (Hoff, 2005)
- flexible: model directed and undirected networks in continuous time

Prior Specification

Assume independent Gaussian process:

$$\mu(\cdot) \sim GP(0, c_\mu), \quad c_\mu(t,t') = exp\{-k_\mu(t-t')^2\}$$

$$\beta_p(\cdot) \sim GP(0, c_p), \quad c_p(t,t') = exp\{-k_p(t-t')^2\}$$

Assume temporal dependence and within-unit dependence, but independence across units:

$$\{a_i(t_1), \ldots, a_i(t_N), b_i(t_1), \ldots, b_i(t_N)\}^T \sim N_{2N}(0, \begin{pmatrix} c_a & \rho_{ab} c_{ab} \\ \rho_{ab} c_{ab} & c_b \end{pmatrix})$$

Introduce a shrinkage parameter to learn the dimension of latent space:

$$\{x_{ih}^s(t_1), \ldots, x_{ih}^s(t_N), x_{ih}^r(t_1), \ldots, x_{ih}^r(t_N)\}^T \sim N_{2N}(0, \tau_h^{-1} \begin{pmatrix} c_{x^s} & \rho_x c_{x^s x^r} \\ \rho_x c_{x^s x^r} & c_{x^r} \end{pmatrix})$$

$$\tau_h = \prod_{l=1}^{h} \nu_l, \quad \nu_1 \sim Ga(a_1, 1), \quad \nu_l \sim Ga(a_2, 1)$$

Common length-scale:

$$c_a(t,t') = c_b(t,t') = c_{ab}(t,t') = exp\{-k_{ab}(t,t')^2\}$$

$$c_{x^s}(t,t') = c_{x^r}(t,t') = c_{x^s x^r}(t,t') = exp\{-k_x(t,t')^2\}$$

Posterior Calculation

We apply Gibbs sampling scheme with Pólya-gamma data augmentation proposed by Polson et al.(2013). We update latent effects jointly within a unit.

SIMULATION

Simulation Settings

We consider a dynamic network with 15 nodes and 40 time points. For true generating process, we set all the length scales to 0.01 and correlations to 0.5, and 2-dimension multiplicative effect. We leave out the network at time 40 to evaluate the performance of prediction.

For prior setting, we consider length scale 0.05, 10-dimension multiplicative effect, and shrinkage parameter to be 2. We ran 5000 iteration discarding the first 1000 as burn-in.

Results

- Great performance on model fitting and prediction. (AUC:0.93,0.78)
- Effective shrinkage method. $(\hat{\tau}_1^{-1}, \hat{\tau}_2^{-1} \approx 0.91, \hat{\tau}_{3:10}^{-1} < 0.27)$
- Without considering network structure, a biased estimation on coefficients and a wider credible interval

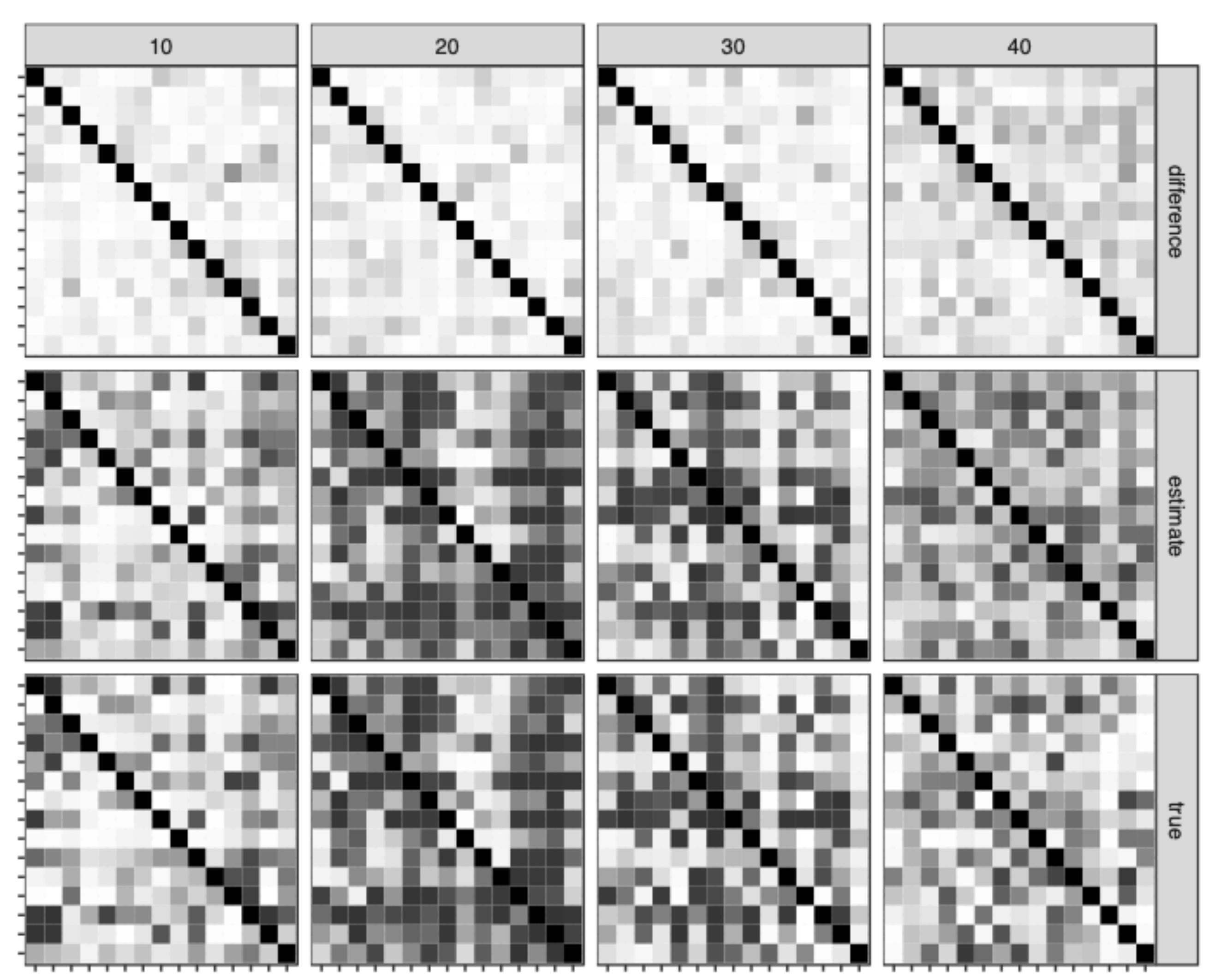

Fig 2 Model performance at selected time.

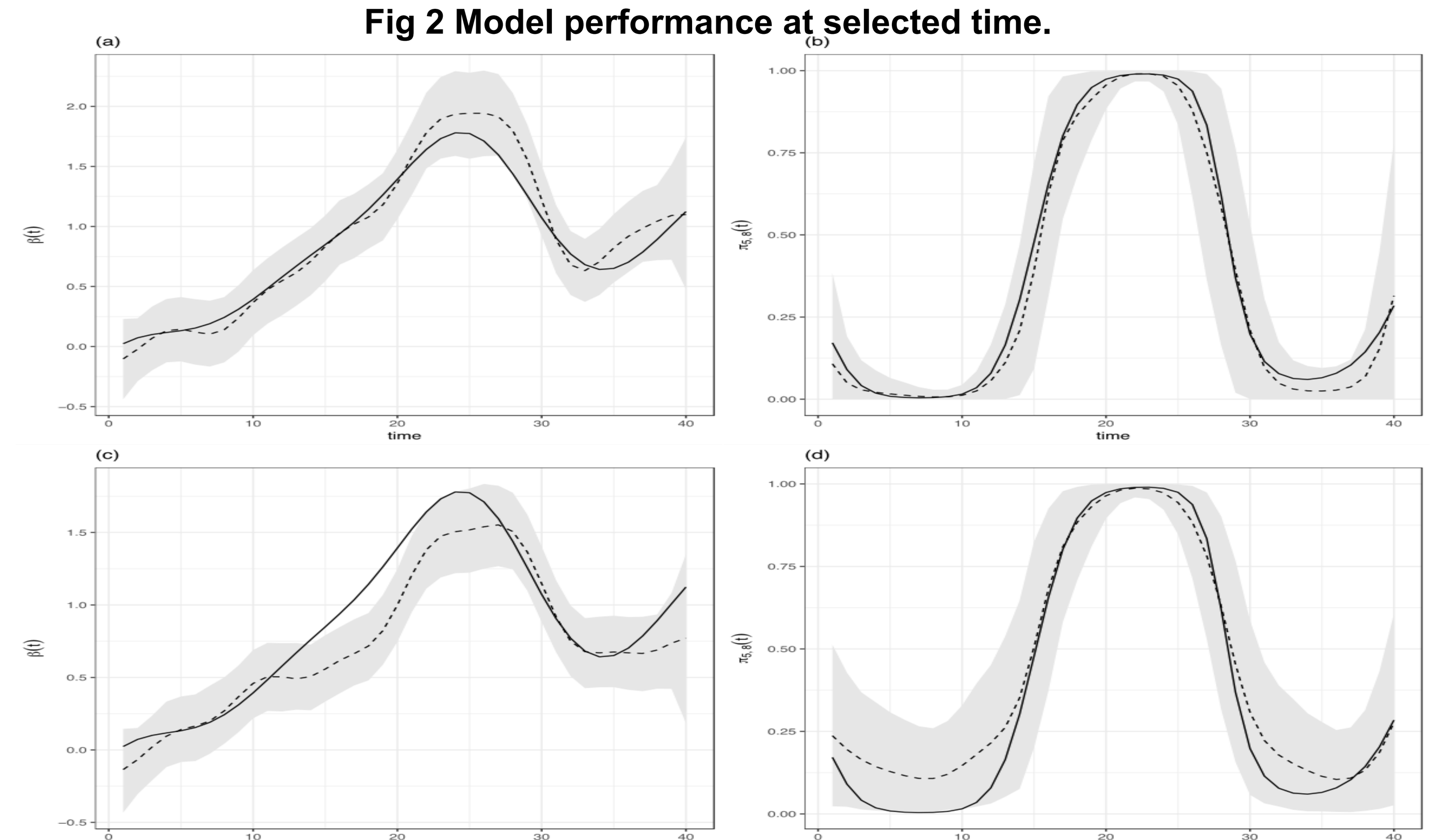

Fig 3 Model comparison between model with and without network structure.

APPLICATION

- We analysis quarterly visiting activities between the world's top 18 largest economies from 2007 to 2016.
- Model formulation:

$$S_{ij}(t) = \mu(t) + \log \mathrm{GDP}_{i,t-1}\beta_1(t) + \log \mathrm{GDP}_{j,t-1}\beta_2(t) + x_i^{sT}(t) x_j^r(t) + a_i(t) + b_j(t)$$

- We ran 50,000 iterations discarding first 5000 posterior samples, and saved samples every 10 iteration.

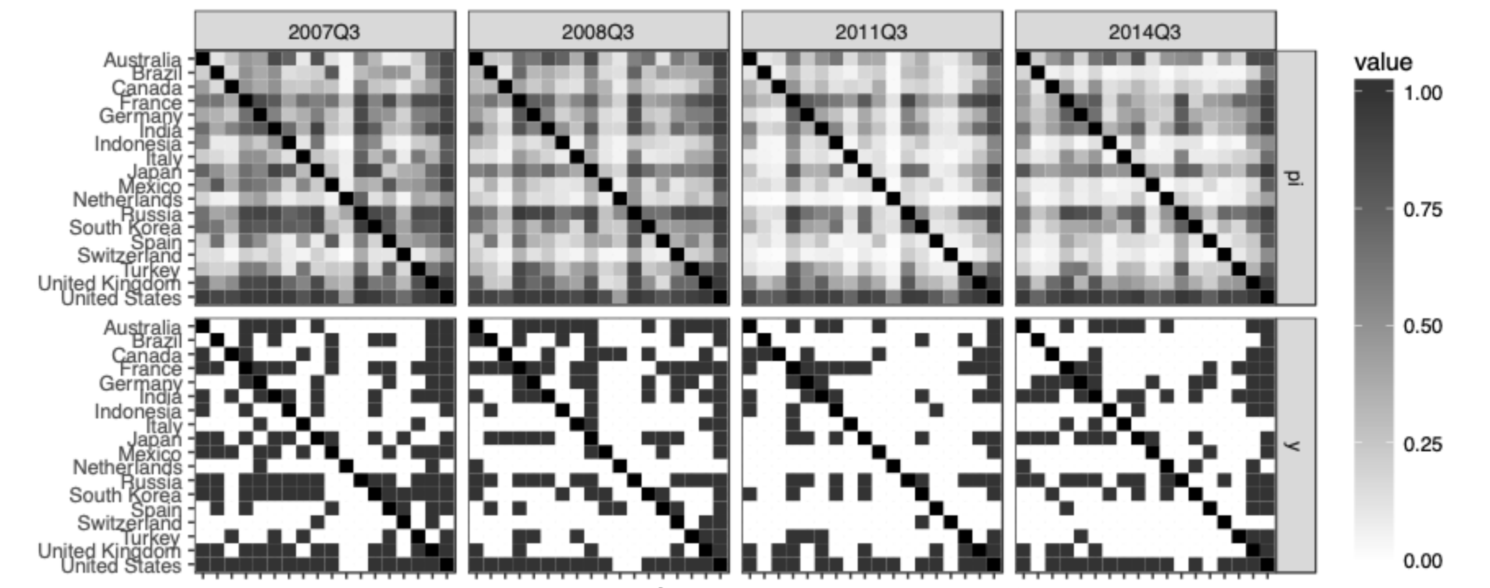

Fig 4 Model performance at selected time.

- Great performance on model fitting and prediction. (AUC:0.88,0.77)
- Countries with a higher GDP prefer making a visit instead of hosting a visit. But the economics' influence is insignificant.

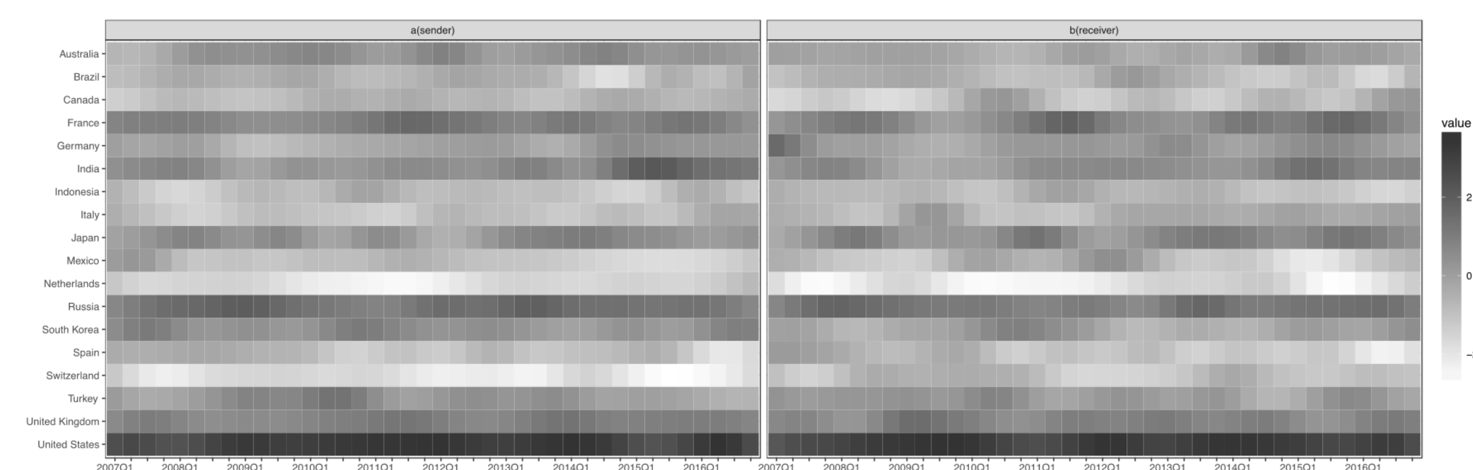

Fig 5 Parameter estimation for addictive effects.

- Reflect role of countries: United States, Russia, the United Kingdoms, France, and Japan played active role. Switzerland hosted several visits but rarely visited other countries. India was involved more in diplomatic relationships since 2014Q2 (Narendra Modi came to power, economics growth).
- Trends of the addictive effects are close related to natural disasters (floods and earthquakes), financial crisis, new administrations, contests, terrorism and other events such as the Olympics.

Example: Japan

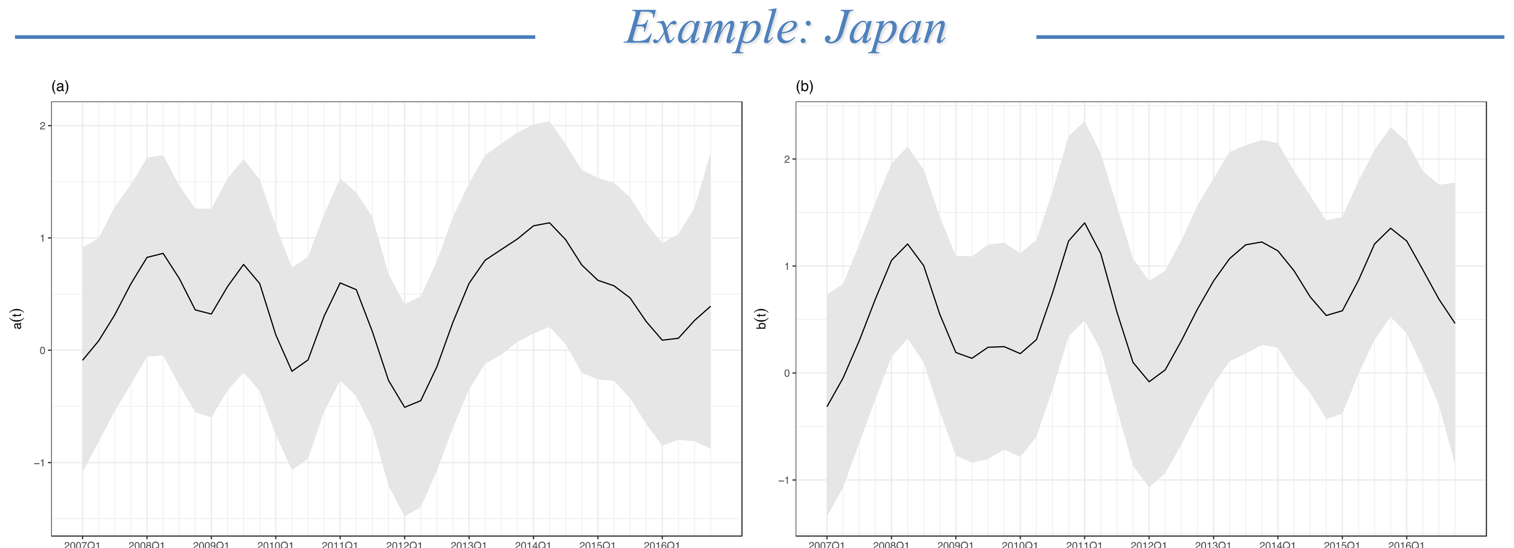

Fig 6 Parameter estimation for addictive effects of Japan.

- Characteristics captured by the model correspond to the events: financial crisis (2008Q2), regime alteration (2009-2011), Fukushima Daiichi nuclear disaster (2011Q2), Abe's return and the economics recovery(2012Q1), the second recession (2014Q2).
- 'Popularity' shows more sensitivity to the economics status, and less sensitivity to the regime alternation.